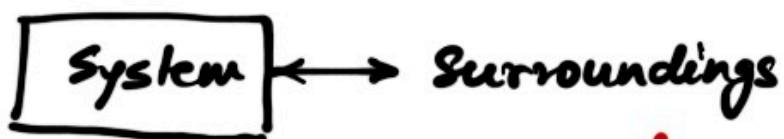


WORK AND ENERGY



Temperature difference
causes the flow of
heat (q)

Heat input into the system $\rightarrow q > 0$

Heat removed from the system $\rightarrow q < 0$

If the energy of the system is increased
 $\rightarrow$ work done on the system
(w) > 0

If the energy of the system is decreased
 $\rightarrow$ work done by the system
 $w < 0$

Pressure-Volume work

- (1) Consider a gas contained in a cylinder
- (2) A piston exerts a force $= mg$

(3) The internal pressure of the gas is P_i . To move the piston outward by a distance h
The work done by the system:

$$W = -mgh$$

This can be rewritten as

$$W = - \frac{mghA}{A}$$

area

Volume

$$= -P_{\text{ext}} \Delta V$$

change in
volume

$$\text{final volume} \longrightarrow V_f - V_i$$

initial volume

When $\Delta V > 0$, expansion, $W < 0$

Expansion occurs until $P_f = P_{\text{ext}}$

When $\Delta V < 0$, compression, $W > 0$

occurs when $P_{\text{ext}} > P_i$

The general expression for work

$$w = - \int_{V_1 \leftarrow \text{initial volume}}^{V_2 \leftarrow \text{final volume}} P_{\text{ext}}(V) dV$$

When $P_{\text{ext}}(V)$ is constant $\rightarrow P_{\text{ext}}$

$$\begin{aligned} w &= - P_{\text{ext}} \int_{V_1}^{V_2} dV \\ &= - P_{\text{ext}} [V_2 - V_1] = - P_{\text{ext}} \Delta V \end{aligned}$$

Example

Consider an ideal gas that occupies a volume of 1 dm^3 at a pressure of 2.0 bar .

If the gas is compressed isothermally at a constant pressure (P_{ext}) so that the final volume is 0.5 dm^3 . What is the smallest P_{ext} value possible? What is the corresponding work involved?

An ideal gas at constant T

$$P_i V_i = P_f V_f \quad (\text{Boyle's Law})$$

$$P_f = \frac{P_i V_i}{V_f}$$

$$= \frac{2 \text{ bar} \times 1 \text{ dm}^3}{0.5 \text{ dm}^3}$$

$$= \underline{4 \text{ bar}}$$

$$W = -P_{\text{ext}} (V_f - V_i)$$

$$= -(4 \text{ bar})(0.5 \text{ dm}^3 - 1.0 \text{ dm}^3)$$

$$= 2 \text{ bar dm}^3$$

$$= (2 \text{ bar dm}^3) (10^5 \text{ Pa/bar}) (10^{-3} \text{ m}^3 / \text{dm}^3)$$

$$= 200 \text{ Pa m}^3 = \underline{200 \text{ J}}$$

State function $\longrightarrow$ A property that only depends on the state of the system independent of how the state was arrived at.

Internal energy (U) is a state function

$$\int_1^2 dU = U_2 - U_1 = \Delta U$$

The difference only depends on the initial and final states irrespective of the path taken to arrive from state 1 to state 2.

Work (w) and heat (q) are path functions.

$$w = - \int_1^2 P_{\text{ext}} dV$$

Must be greater than the pressure of the gas to be able to compress it

A minimum amount of work is required when P_{ext} is infinitesimally larger than P .

→ System is at equilibrium at all times

→ Reversible process

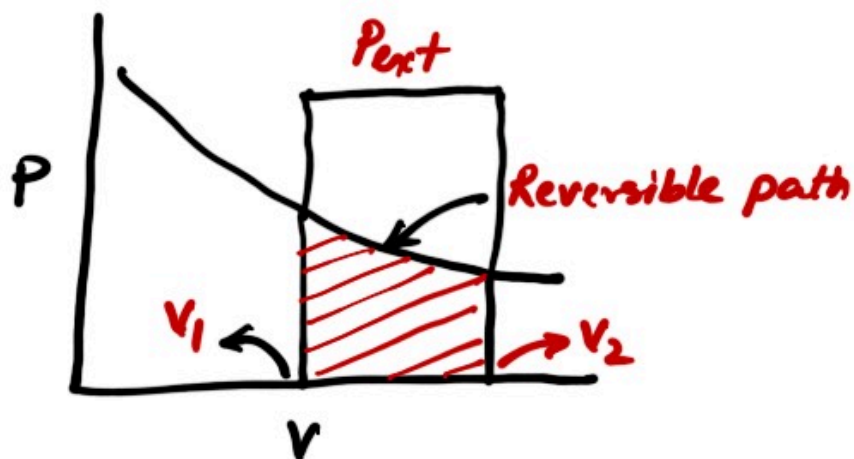
For a reversible process

$$P_{\text{ext}} \approx P$$
$$W_{\text{rev}} = - \int_1^2 P dV$$

Thus, for an ideal gas

$$W_{\text{rev}} = - \int_1^2 \frac{nRT}{V} \cdot dV$$
$$= -nRT \ln\left(\frac{V_2}{V_1}\right)$$

Since we are compressing the gas,
 $V_1 > V_2$, $W_{\text{rev}} > 0$

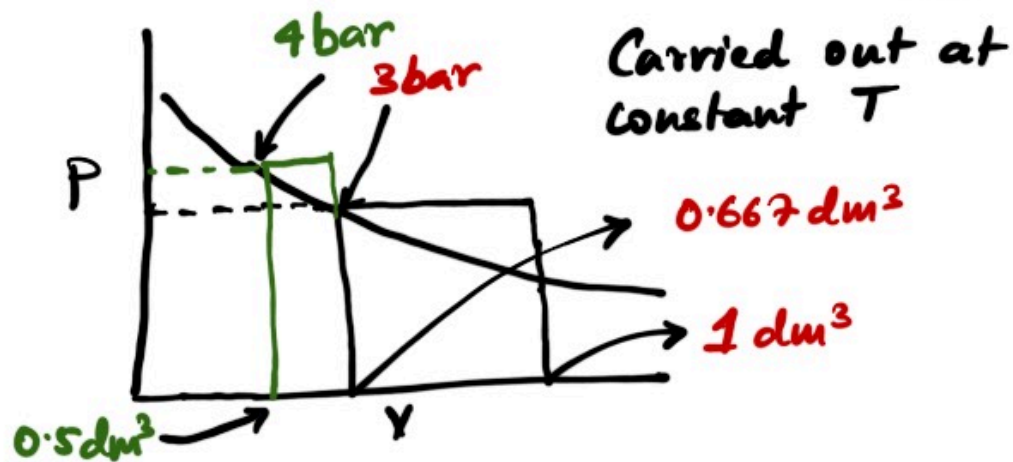


Constant pressure compression in two stages

- (1) Compression from 1 dm^3 to 0.667 dm^3
- (2) Compression from 0.667 dm^3 to 0.5 dm^3

Pressure during compression 1: 3 bar
 $= 3 \times 10^5 \text{ Pa} = 3.5 \times 10^5 \text{ Nm}^{-2}$

Pressure during compression 2: 4 bar
 $= 4 \times 10^5 \text{ Pa} = 4 \times 10^5 \text{ Nm}^{-2}$



$$\begin{aligned}
 W_1 \text{ (work during the first compression)} \\
 &= - \underset{\text{Nm}^{-2}}{3 \times 10^5} \times (\underset{\text{m}^3}{0.667 - 1.00}) \times 10^{-3} \\
 &= 99.9 \text{ Nm} = 99.9 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 W_2 \text{ (work during the second compression)} \\
 &= - 4 \times 10^5 \times (0.5 - 0.667) \times 10^{-3} \\
 &= 66.8 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \text{Total work} &= 99.9 + 66.8 \\
 (W) &= \underline{166.7 \text{ J}}
 \end{aligned}$$

For reversible isothermal compression
from 1.0 dm^3 to 0.5 dm^3

$$W_{\text{rev}} = - nRT \ln \frac{V_2}{V_1}$$

$$\text{Final pressure} = 4.0 \times 10^5 \text{ Pa}$$

$$\begin{aligned}
 \text{So } nRT &= P_1 V_1 = P_2 V_2 \\
 &= 0.5 \times 10^{-3} \times 4.0 \times 10^5 \\
 &= 200 \text{ m}^3 \text{ Pa}
 \end{aligned}$$

$$W_{\text{rev}} = - 200 \ln \left(\frac{0.5}{1.0} \right) = \underline{138.6 \text{ J}}$$

$$W_{\text{rev}} < W$$

So the work done for the reversible compression of the gas is the minimum amount of work done on the gas.

Clearly work is NOT A STATE FUNCTION
it is a PATH FUNCTION.

Energy is a state function (First Law of Thermodynamics)

$$\int_1^2 \delta W = W \neq W_2 - W_1 \quad \text{since}$$

it depends on the path
taken from $1 \rightarrow 2$

$\delta W \rightarrow$ inexact differential

$$\int_1^2 dU = U_2 - U_1$$

$dU \rightarrow$ exact differential

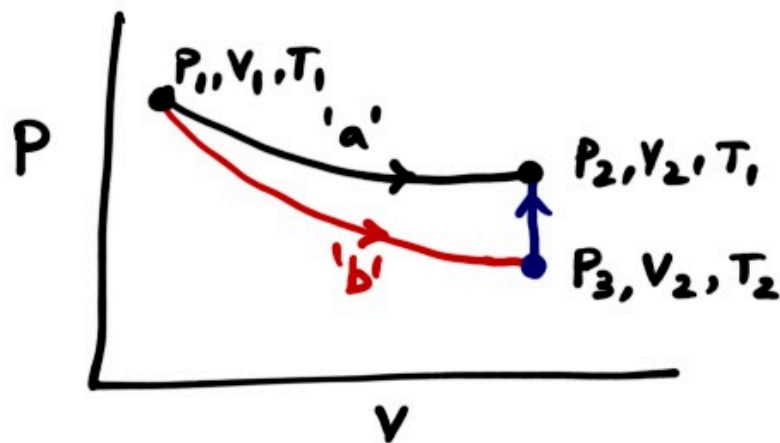
$$\int_1^2 \delta q = q \neq q_2 - q_1$$

$\delta q \rightarrow$ inexact differential

$$\begin{aligned} dU &= \delta q + \delta w \\ \Delta U &= q + w \end{aligned} \left. \vphantom{\begin{aligned} dU &= \delta q + \delta w \\ \Delta U &= q + w \end{aligned}} \right\} \text{Conservation of energy}$$

A statement of the First Law of Thermodynamics

Consider the transformation of an ideal gas from an initial state (P_1, V_1, T_1) to a final state (P_2, V_2, T_1) . This is an isothermal change.



Use path 'a' or use paths 'b' and 'c'.

Since the final state is the same in both cases, ΔU is the same

The energy of an ideal gas depends only on T

$$\begin{aligned}\Delta U_a &= 0 = \delta q_{a,rev} + \delta w_{a,rev} \\ -\delta q_{a,rev} &= \delta w_{a,rev} \\ &= -\int_1^2 P \cdot dV \\ &= -\int_1^2 \frac{RT_1}{V} dV \\ &= -RT \ln\left(\frac{V_2}{V_1}\right)\end{aligned}$$

Since $V_2 > V_1$, $w_{rev} < 0$, $q_{rev} > 0$

Path 'b' $\rightarrow$ reversible expansion from (P_1, V_1, T_1) to (P_3, V_2, T_2)

this change is done adiabatically i.e. there is no exchange of heat with the surroundings.

Path 'c' involves changing the temperature from T_2 to T_1 at constant volume.

We define a new quantity $C_V \rightarrow$
heat capacity at constant volume

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V$$

Measures amount of energy required to change the temperature by 1 K at constant volume $\rightarrow \text{JK}^{-1}$

For an ideal gas, path 'b' can be recast terms of a change in T from T_1 to T_2 .

Since,

$$\begin{aligned} q_{b,\text{rev}} &= 0 & \Delta U_b &= w_{b,\text{rev}} \\ &= \int_{T_1}^{T_2} \left(\frac{\partial U}{\partial T} \right)_V dT = \int_{T_1}^{T_2} C_V(T) dT \end{aligned}$$

For path 'c', there is no pressure-volume work involved since there is no volume change (V_2)

$$q_{c, \text{rev}} = \Delta U_c = \int_{T_2}^{T_1} C_v(T) dT$$

$$\begin{aligned} q_{bc, \text{rev}} &= q_{b, \text{rev}} + q_{c, \text{rev}} \\ &= 0 + \int_{T_2}^{T_1} C_v(T) dT \\ &= \int_{T_2}^{T_1} C_v(T) dT \end{aligned}$$

The work is then

$$\begin{aligned} w_{bc, \text{rev}} &= w_{b, \text{rev}} + w_{c, \text{rev}} \\ &= \int_{T_1}^{T_2} C_v(T) dT + 0 \end{aligned}$$

Thus,

$$\begin{aligned}\Delta U_{BC} &= \Delta U_B + \Delta U_C \\ &= \int_{T_1}^{T_2} C_V(T) dT + \int_{T_2}^{T_1} C_V(T) dT \\ &= 0\end{aligned}$$

This is the same as path 'a' but the work (w) and heat (q) changes are different in the two cases: 'a' and 'b, c'.

Consider a reversible adiabatic expansion

$$dU = \delta w \quad \text{since } \delta q = 0$$

δw in this case is an exact differential

$$dU = C_V(T) dT$$

$$dw = -P dV = -\frac{nRT}{V} dV$$

Thus,

$$C_V(T) dT = -\frac{nRT}{V} dV$$

Defining $\frac{C_V(T)}{n} = \bar{C}_V(T) \rightarrow$ molar heat capacity

$$\int_{T_1}^{T_2} \frac{\bar{C}_V(T)}{T} dT = -R \int_{V_1}^{V_2} \frac{dV}{V}$$

For a ideal monoatomic gas

$$\bar{C}_V(T) = \frac{3}{2}R \quad \text{i.e. it is independent of } T$$

$$\frac{3R}{2} \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\frac{3}{2} \ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{V_1}{V_2}\right)$$

Thus,

$$\left(\frac{T_2}{T_1}\right)^{3/2} = \frac{V_1}{V_2}$$

For an adiabatic process if $V_2 > V_1$
i.e for an expansion, $T_1 > T_2$

The system is cooled for an adiabatic expansion.

Also,

$$\left(\frac{P_2 V_2}{P_1 V_1} \right)^{3/2} = \frac{V_1}{V_2}$$

$$P_1 V_1^{5/3} = P_2 V_2^{5/3}$$

Shows the relation between pressure and volume for an adiabatic process.

For an isothermal process

$$P_1 V_1 = P_2 V_2 \quad \text{Boyle's Law}$$

For a reversible process with only pressure-volume work

$$\Delta U = q + w = q - \int_{V_1}^{V_2} P dV$$

At constant volume

$$\Delta U_v = q_v$$

Energy can be measured by measuring the heat change

At constant pressure,

$$\Delta U_p = q_p - P_{\text{ext}} \int_{V_1}^{V_2} dV$$

$$q_p = \Delta U_p + P_{\text{ext}} \Delta V = \Delta(U + PV)$$

A new state
function (H)

enthalpy

$$\Delta H = q_p$$

$$\Delta U = q_v \rightarrow \text{heat change at constant volume}$$

$$\Delta H = q_p \rightarrow \text{heat change at constant pressure}$$

$C_p \rightarrow$ Heat capacity at constant pressure

$$C_p = \left(\frac{\partial H}{\partial T} \right)_p \quad \text{JK}^{-1}$$

For an ideal gas

$$H = U + nRT$$

Thus,

$$\left(\frac{\partial H}{\partial T}\right)_P = \left(\frac{\partial U}{\partial T}\right)_V + nR$$

Since P, V no longer appear in the equation

$$C_p = C_v + nR$$

Enthalpy change for a chemical reaction (ΔH_r)

$$\Delta H_r = H_{\text{products}} - H_{\text{reactants}}$$

Endothermic $\rightarrow$ Heat absorbed $\rightarrow$

$$H_{\text{products}} > H_{\text{reactants}}$$

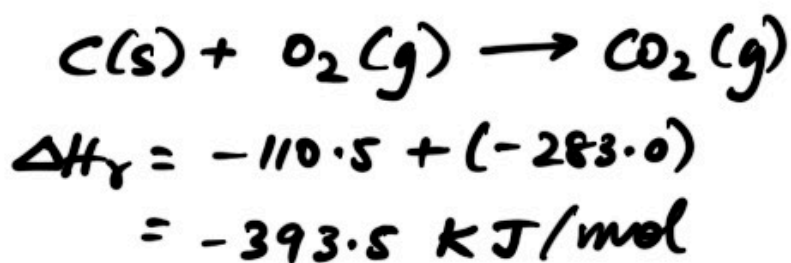
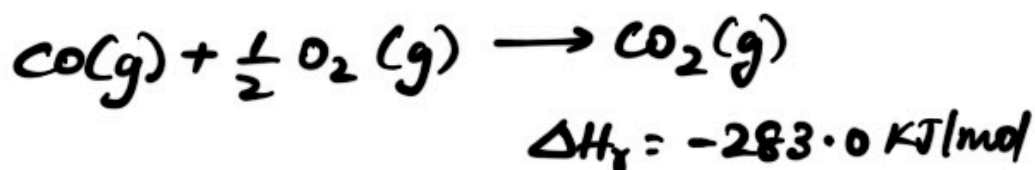
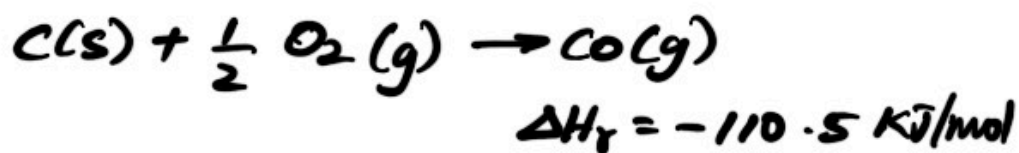
$$\Delta H_r > 0$$

Exothermic $\rightarrow$ Heat released

$$H_{\text{products}} < H_{\text{reactants}}$$

$$\Delta H_r < 0$$

Additivity of ΔH_r



$\Delta H_r < 0$ Exothermic

Additivity of $\Delta H_r \rightarrow$ Hess's Law

$$\Delta H_{r, \text{forward}} = -\Delta H_{r, \text{reverse}}$$

Consider the following chemical reaction



Let us express the enthalpies of reaction at $T = T_2$ as

$$\begin{aligned}\Delta H_r(T_2) = & y [H_Y(T_2) - H_Y(0)] \\ & + z [H_Z(T_2) - H_Z(0)] \\ & - a [H_A(T_2) - H_A(0)] \\ & - b [H_B(T_2) - H_B(0)]\end{aligned}$$

where

$$H_i(T_2) - H_i(0) = \int_0^{T_2} C_{p,i}(T) dT$$

$i = A, B, Y, Z$

Similarly, at $T = T_1$,

$$\begin{aligned}\Delta H_r(T_1) = & y [H_Y(T_1) - H_Y(0)] \\ & + z [H_Z(T_1) - H_Z(0)] \\ & - a [H_A(T_1) - H_A(0)] \\ & - b [H_B(T_1) - H_B(0)]\end{aligned}$$

where

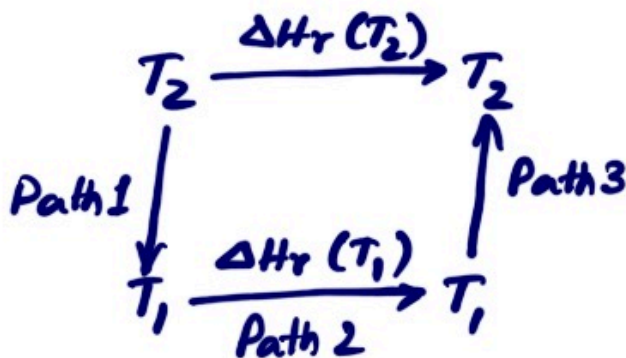
$$H_i(T_1) - H_i(0) = \int_0^{T_1} C_{p,i}(T) dT$$

Subtracting the expression for $\Delta H_r(T_1)$ from that for $\Delta H_r(T_2)$

$$\Delta H_r(T_2) - \Delta H_r(T_1) = \int_{T_1}^{T_2} \Delta C_p(T) dT$$

where

$$\Delta C_p(T) = y C_{p,Y}(T) + z C_{p,Z}(T) - a C_{p,A}(T) - b C_{p,B}(T)$$



Following the path $1 \rightarrow 2 \rightarrow 3$, we take the reactants from $T_2 \rightarrow T_1$, carry out the reaction at T_1 , and then take the products from $T_1 \rightarrow T_2$

$$\Delta H_1 = \int_{T_2}^{T_1} C_p(T)_{\text{reactants}} dT \quad T_2 \rightarrow T_1$$

$$= - \int_{T_1}^{T_2} C_p(T)_{\text{reactants}} dT$$

Reaction at T_1

$$\Delta H_2 = \Delta H_r(T_1)$$

$$\Delta H_3 = \int_{T_1}^{T_2} C_p(T)_{\text{products}} dT \quad T_1 \rightarrow T_2$$

$$\Delta H_r(T_2) = \Delta H_1 + \Delta H_2 + \Delta H_3$$

$$= \Delta H_r(T_1)$$

$$+ \int_{T_1}^{T_2} [C_p(T)_{\text{products}} - C_p(T)_{\text{reactants}}] dT$$

PROBLEMS

- (1) Derive an expression for the reversible, isothermal work on a Van der Waals gas from a state 1 to a state 2.

$$W_{\text{rev}} = - \int_1^2 P dV$$

For a Van der Waals gas

$$P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$$

(Note that we are looking at volumes, not molar volumes)

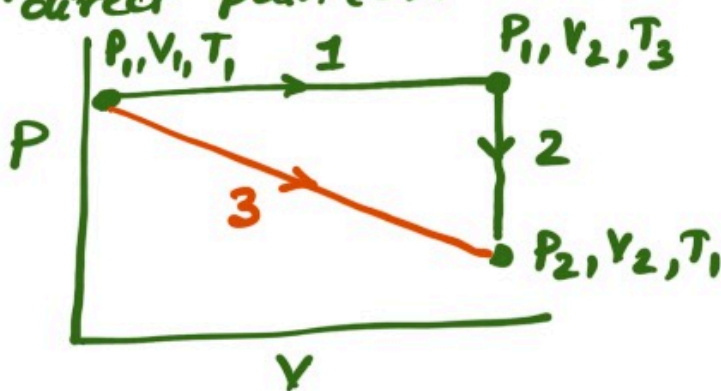
$$\begin{aligned} W_{\text{rev}} &= -nRT \int_1^2 \frac{dV}{V-nb} + an^2 \int \frac{dV}{V^2} \\ &= -nRT \ln \left[\frac{V_2 - nb}{V_1 - nb} \right] \\ &\quad + an^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right] \end{aligned}$$

When $a=0$, $b=0$

$$W_{\text{rev}} = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

which the expression for an ideal gas

- (2) Calculate the change in internal energy for the 2-step process shown below. Also calculate the q_{rev} , w_{rev} for each step. How do these values compare with the direct path (3)?



Path 1 carried out at constant pressure

$$\Delta U_1 = \int_{T_1}^{T_3} C_V(T) dT$$

$$W_{\text{rev},1} = -P_1(V_2 - V_1)$$

$$q_{rev,1} = \int_{T_1}^{T_3} C_V(T) dT + P_1 (V_2 - V_1)$$

Path 2

Constant volume

$$\Delta U_2 = \int_{T_3}^{T_1} C_V(T) dT = - \int_{T_1}^{T_3} C_V(T) dT$$

No volume change, $w_{rev,2} = 0$

$$q_{rev,2} = - \int_{T_1}^{T_3} C_V(T) dT$$

$$\begin{aligned} \Delta U_{total} &= \Delta U_1 + \Delta U_2 \\ &= \int_{T_1}^{T_3} C_V(T) dT - \int_{T_1}^{T_3} C_V(T) dT \\ &= \underline{0} \end{aligned}$$

Not surprising, since the temperatures of the two states are the same.

$$\begin{aligned}
 q_{\text{total}} &= q_{\text{rev},1} + q_{\text{rev},2} \\
 &= \int_{T_1}^{T_3} C_V(T) dT + P_1(V_2 - V_1) \\
 &\quad - \int_{T_1}^{T_3} C_V(T) dT \\
 &= \underline{P_1(V_2 - V_1)}
 \end{aligned}$$

$$\begin{aligned}
 w_{\text{total}} &= w_{\text{rev},1} + w_{\text{rev},2} \\
 &= -P_1(V_2 - V_1) + 0 \\
 &= \underline{-P_1(V_2 - V_1)}
 \end{aligned}$$

For path 3, we have an isothermal (at T_1) expansion from (P_1, V_1) to (P_2, V_2)

$$\Delta U_3 = 0$$

$$\delta q_{\text{rev},3} = -\delta w_{\text{rev},3} = -P dV$$

$$q_{\text{rev},3} = -w_{\text{rev},3} = \underline{-RT \ln \frac{V_2}{V_1}}$$

(3) given the heats of formation

$$\text{C}_2\text{H}_4(\text{g}) = 52.28 \text{ KJ/mol}$$

$$\text{C}_2\text{H}_5\text{OH}(\text{l}) = -277.69 \text{ KJ/mol}$$

$$\text{H}_2\text{O}(\text{g}) = -241.8 \text{ KJ/mol}$$

The heat of vaporization of H_2O
 $= 44.03 \text{ KJ/mol}$

Calculate the enthalpy change for the following reaction:



$$\Delta H_r = \Delta H_{\text{products}} - \Delta H_{\text{reactants}}$$

$$= -277.69 - 52.28 + 241.8 + 44.03$$

$$\begin{array}{l} \text{l} \rightarrow \text{g} + 44.03 \\ \text{g} \rightarrow \text{l} - 44.03 \end{array}$$

$$= -44.14 \text{ KJ/mol}$$

Exothermic reaction

(4) Show that for a gas that obeys the following equation of state

$$P(\bar{V}-b) = RT$$

The following expression holds for a reversible adiabatic expansion

$$\left(\frac{T_2}{T_1}\right)^{\bar{C}_V/R} = \frac{\bar{V}_1 - b}{\bar{V}_2 - b}$$

Assume that $\bar{C}_V$ is independent of T .

What happens when $\bar{C}_V = \frac{3R}{2}$?

For a reversible adiabatic process,

$$q_{\text{rev}} = 0 \quad \text{and} \quad w_{\text{rev}} = \Delta U$$

$$\Delta U = n \int_{T_1}^{T_2} \bar{C}_V dT$$

$$w_{\text{rev}} = -n \int_{\bar{V}_1}^{\bar{V}_2} P d\bar{V}$$

$$\text{Thus,} \quad \int_{T_1}^{T_2} \bar{C}_V dT = - \int_{\bar{V}_1}^{\bar{V}_2} P d\bar{V}$$

using the expression for P from the equation of state

$$P = \frac{RT}{\bar{V} - b}$$

$$\int_{T_1}^{T_2} \frac{\bar{C}_v dT}{T} = -R \int_{\bar{V}_1}^{\bar{V}_2} \frac{d\bar{V}}{\bar{V} - b}$$

Since $\bar{C}_v$ is independent of T

$$\bar{C}_v \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{\bar{V}_1}^{\bar{V}_2} \frac{d\bar{V}}{\bar{V} - b}$$

$$\bar{C}_v \ln\left(\frac{T_2}{T_1}\right) = -R \ln\left[\frac{\bar{V}_2 - b}{\bar{V}_1 - b}\right]$$

$$\left(\frac{T_2}{T_1}\right)^{\bar{C}_v/R} = \frac{\bar{V}_1 - b}{\bar{V}_2 - b}$$

When $\bar{C}_V = 3R/2$

$$\underline{\left(\frac{T_2}{T_1}\right)^{3/2} = \frac{\bar{V}_1 - b}{\bar{V}_2 - b}}$$

(5) Derive a general relationship between C_p and C_v . From the general relationship obtain one for an ideal gas.

$$U = f(P, T)$$

$$dU = \left(\frac{\partial U}{\partial P}\right)_T dP + \left(\frac{\partial U}{\partial T}\right)_P dT$$

Similarly,

$$U = f(V, T)$$

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$$

Also

$$V = f(P, T)$$

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP$$

Thus,

$$dU = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T dP + \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial U}{\partial T}\right)_V dT$$

Comparing with the first equation

$$\left(\frac{\partial U}{\partial P}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T$$

$$\left(\frac{\partial U}{\partial T}\right)_P = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P + \underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{C_V}$$

Now,

$$H = U + PV$$

$$\underbrace{\left(\frac{\partial H}{\partial T}\right)_P}_{C_P} = \left(\frac{\partial U}{\partial T}\right)_P + P \left(\frac{\partial V}{\partial T}\right)_P$$

$$\begin{aligned}
 C_p &= \left(\frac{\partial U}{\partial T} \right)_p + p \left(\frac{\partial V}{\partial T} \right)_p \\
 &= \left(\frac{\partial U}{\partial V} \right)_T \left(\frac{\partial V}{\partial T} \right)_p + \left(\frac{\partial U}{\partial T} \right)_V \\
 &\quad + p \left(\frac{\partial V}{\partial T} \right)_p
 \end{aligned}$$

$$\begin{aligned}
 &= \left(\frac{\partial U}{\partial V} \right)_T \left(\frac{\partial V}{\partial T} \right)_p + C_v \\
 &\quad + p \left(\frac{\partial V}{\partial T} \right)_p
 \end{aligned}$$

$$C_p - C_v = \left(\frac{\partial U}{\partial V} \right)_T \left(\frac{\partial V}{\partial T} \right)_p + p \left(\frac{\partial V}{\partial T} \right)_p$$

For an ideal gas,

$$\left(\frac{\partial U}{\partial V} \right)_T = 0$$

$$C_p - C_v = p \left(\frac{\partial V}{\partial T} \right)_p$$

For an ideal gas, $PV = nRT$

$$P\left(\frac{\partial V}{\partial T}\right) = nR$$

$$\underline{C_p - C_v = nR}$$

(6) The quantity $\left(\frac{\partial T}{\partial P}\right)_H$ is known as the Joule-Thompson co-efficient. What is its value for an ideal gas assuming an isenthalpic process?

$$H = U + PV \quad H \rightarrow f(P, T)$$

$$dH = \left(\frac{\partial H}{\partial P}\right)_T dP + \left(\frac{\partial H}{\partial T}\right)_P dT$$

$$= \left(\frac{\partial H}{\partial P}\right)_T dP + C_p dT$$

For an isenthalpic process

$$dH = 0$$

$$\left(\frac{\partial H}{\partial P}\right)_T dP = -C_p dT$$

$$C_p \left(\frac{\partial T}{\partial P} \right)_H = - \left(\frac{\partial H}{\partial P} \right)_T$$

$$\text{Thus, } \mu_{JT} = \left(\frac{\partial T}{\partial P} \right)_H = \frac{1}{C_p} \left(\frac{\partial H}{\partial P} \right)_T$$

$$H = U + PV$$

$$\left(\frac{\partial H}{\partial P} \right)_T = \left(\frac{\partial U}{\partial P} \right)_T + \left(\frac{\partial PV}{\partial P} \right)_T$$

For an ideal gas,

$$\left(\frac{\partial U}{\partial P} \right)_T = 0$$

$$\left(\frac{\partial}{\partial P} \right)_T (PV) = \left(\frac{\partial}{\partial P} \right)_T (nRT)$$

=

Thus, for an ^o ideal gas

$$\underline{\mu_{JT} = 0}$$